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# Common Fixed Point Theorems of Compatible Mapping of Type (P) in Intuitionistic Fuzzy Metric Spaces

***Rajesh Shrivastava**, *Ramakant Bhardwaj*** and *Vipin Kumar Sharma*******

**Professor, Department of Mathematics, Excellence College, Bhopal, (Madhya Pradesh), INDIA*
***Professor, Department of Mathematics, TIT College, Bhopal, (Madhya Pradesh), INDIA*
****Assistant Professor, Department of Mathematics, LNCT College, Bhopal, (Madhya Pradesh), INDIA*

*(Corresponding author: Vipin Kumar Sharma)*
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**ABSTRACT: In this paper we prove a common fixed point theorem in intuitionistic fuzzy metric spaces using the notion of weakly compatible. B.P. Tripathi, G.S. Saluja, D.P. Sahu and N. Namdeo [21] point out results of M. Koireng and Yumnam Rahen [10] on compatible mappings of type (P) in fuzzy metrics paces into intuitionistic fuzzy metric spaces with same terminology and notations. In this paper we generalized the result of B.P. Tripathi, G.S. Saluja, D.P. Sahu and N. Namdeo [21].**

## I. INTRODUCTION

Fuzzy set theory was introduced by Zadeh in 1965 [24]. Many authors haveintroduced and discussed several notions of fuzzy metric space in different ways[11], [5], [6] and also proved fixed point theorems with interesting consequent results in the fuzzy metric spaces [7]. Recently the concept of intuitionistic fuzzy metric space was given by Park [13] and the subsequent fixed point results in the intuitionistic fuzzy metric spaces are investigated by Alaca et al. [1] and Mohamad [12] (see, also [2], [3], [18] and [23]).

## II. PRELIMINARIES

The study of fixed points of various classes of mappings have been the focus of vigorous research for many Mathematicians. Among them one of the important result in theory of fixed points of compatible mappings was obtained by G. Jungck [8] in 1986. Since then there have been a flood of research papers involving various types of compatibility such as Compatible mappings of type (A)[9], Semi-compatibility [4], compatible mappings of type (P) [14], compatible mappings of type (B) [16] and compatible mappings of type (C) [17] etc. The following definitions, lemma and examples are useful for our presentations.

**Definition 2.1 (See [19]).** A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is continuoust-norm if the binary operation $*$ satisfying the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, $d \in [0,1]$.

**Definition 2.2 (See [19]).** A binary operation : $[0,1] \times [0,1] \rightarrow [0,1]$ is continuoust-conorm if the binary operation satisfying the following conditions:
(i)$\Diamond$ is commutative and associative,
(ii)$\Diamond$ is continuous,
(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$,
(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d $\in [0,1]$.

**Definition 2.3 (See [1]).** A 5- tuple $(X, M, N, *, \Diamond)$ is called a intuitionistic
fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, is a continuoust-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions: for all x, y, $z \in X$ and $s, t > 0$
(IFM −1) $M(x, y,t) + N(x, y,t) \leq 1$,
(IFM −2) $M(x, y,0) = 0$,
(IFM −3) $M(x, y,t) = 1$ if and only if $x = y$,
(IFM −4) $M(x, y,t) = M(y, x,t)$,
(IFM −5) $M(x, y,t) * M(y,z,s) \leq M(x,z,t +s)$,
(IFM −6) $M(x, y,.): (0,\infty) \rightarrow (0,1]$ is left continuous,
(IFM −7) $\lim_{t\rightarrow\infty} M(x, y,t) = 1$,
(IFM −8) $N(x, y,0) = 1$,
(IFM −9) $N(x, y,t) = 0$ if and only if $x = y$,
(IFM −10) $N(x, y,t) = N(y, x,t)$,
(IFM −11) $N(x, y,t) \Diamond N(y,z,s) \leq N(x,z,t +s)$,
(IFM −12) $N(x, y,.): (0,\infty) \rightarrow (0,1]$ is right continuous,
(IFM −13) $\lim_{t\rightarrow\infty} N(x, y,t) = 0$.
Then (M, N) is called an intuitionistic fuzzy metric on X. The functions
M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

**Remark 2.4**. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metricspace of the form $(X, M, 1-M, *, \lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated, that is, $x \lozenge y = 1-((1-x) * (1-y))$ for all $x, y \in X$.

**Example 2.5.** (Induced intuitionistic fuzzy metric space) Let $(X, d)$ be a metric space. Define $a*b = ab$ and $a \lozenge b = \min\{1, a+b\}$ for all $a, b \in [0,1]$ and let $M_d$ and $N_d$ be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$M_d(x, y,t) = \frac{t}{t+d(x,y)}$,

$N_d(x, y,t) = \frac{d(x,y)}{t+d(x,y)}$.

Then $(X, M_d, N_d, *, \lozenge)$ is an intuitionistic fuzzy metric induced by metric d the standard intuitionistic fuzzy metric space.

**Definition 2.6 (See [1]).** Let $(X, M, N, *, )$ be an intuitionistic fuzzy metric
space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point x in X if and
only if $\lim_{n\to\infty} M(x_n, x,t) = 1$ and $\lim_{n\to\infty} N(x_n, x,t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x,t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x,t) = 0$ for each $p > 0$ and $t > 0$.

(c) An intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be complete
if and only if every Cauchy sequence in X is convergent in X.

**Lemma 2.7 (See [20]).** Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ for all $t \in [0,1]$.

If there exists a number $q \in (0,1)$ such that $M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n,t)$
and $N(x_{n+2}, x_{n+1},qt) \leq N(x_{n+1}, x_n,t)$ for all $t > 0$ and $n \in N$, then $\{x_n\}$ is a
Cauchy sequence in X.

**Proof.** For $t > 0$ and $q \in (0,1)$ we have,

$aM(x_2, x_3,qt) \geq M(x_1, x_2,t) \geq M(x_0, x_1,\frac{t}{q})$,

or

$M(x_2, x_3,t) \geq M(x_0, x_1,\frac{t}{q^2})$.

By simple induction, we have for all $t > 0$ and $n \in N$,

$M(x_{n+1}, x_{n+2},t) \geq M(x_1, x_2,\frac{t}{q^n})$.

Thus for any positive number p and real number $t > 0$, we have

$M(x_n, x_{n+p},t) \geq M(x_n, x_{n+1},\frac{t}{p}) * \ldots\ldots * M(x_{n+p-1}, x_{n+p},\frac{t}{p})$, by (IFM−5),

$\geq M(x_1, x_2,\frac{t}{pq^{n-1}}) * \ldots\ldots * M(x_1, x_2,\frac{t}{pq^{n+p-2}})$.

Therefore by (IFM −7), we have

$M(x_n, x_{n+p},t) \geq 1 * \ldots\cdots * 1 \geq 1$.

Similarly, for $t> 0$ and $q \in (0,1)$, we have

$N(x_2, x_3,qt) \leq N(x_1, x_2,t) \leq N(x_0, x_1,\frac{t}{q})$,

or

$N(x_2, x_3,t) \leq N(x_0, x_1,\frac{t}{q^2})$.

By simple induction, we have for all $t > 0$ and $n \in N$,

$N(x_{n+1}, x_{n+2},t) \leq N(x_1, x_2,\frac{t}{q^n})$.

Thus for any positive number p and real number $t > 0$, we have

$N(x_n, x_{n+p},t) \leq N(x_n, x_{n+1},\frac{t}{p}) \lozenge \ldots\ldots \lozenge N(x_{n+p-1}, x_{n+p},\frac{t}{p})$, by (IFM −11),

$\leq N(x_1, x_2,\frac{t}{pq^{n-1}}) \lozenge \ldots\ldots \lozenge N(x_1, x_2,\frac{t}{pq^{n+p-2}})$.Therefore by (IFM −13), we have

$N(x_n, x_{n+p},t) \leq 0 \lozenge \ldots\ldots \lozenge 0 \leq 0$.

This implies that $\{x_n\}$ is a Cauchy sequence in X. This completes the proof.

**Lemma 2.8 (See [20]).** Let $(X, M, N, *, )$ be an intuitionistic fuzzy metric
space. If $\forall x, y \in X$ and $t > 0$ with positive number $q \in (0,1)$ and $M(x, y,qt) \geq M(x, y,t)$ and $N(x, y,qt) \leq N(x, y,t)$, then $x = y$.

**Proof.** If for all $t > 0$ and some constant $q \in (0,1)$, then we have

$M(x, y,t) \geq M(x, y,\frac{t}{q}) \geq M(x, y,\frac{t}{q^2}) \geq \cdots \geq M(x, y,\frac{t}{q^n}) \geq \ldots$,

and

$N(x, y,t) \leq N(x, y,\frac{t}{q}) \leq N(x, y,\frac{t}{q^2}) \leq \cdots \leq N(x, y,\frac{t}{q^n}) \leq \ldots$,

$n \in N$ and for all $t > 0$ and $x, y \in X$. When $n \to \infty$, we have $M(x, y,t) = 1$
and $N(x, y,t) = 0$ and thus $x = y$. This completes the proof.

**Definition 2.9 (See [22]).** Two self-mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are called compatible if

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ forsome $x \in X$.

Definition 2.10. Two self mappings A and S of an intuitionistic fuzzy metric
space $(X, M, N, *, \lozenge)$ are called compatible of type (P) if

$\lim_{n\to\infty} M(AAx_n, SSx_n, t) = 1$

$\lim_{n\to\infty} (AAx_n, SSx_n, t) = 0$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ forsome $x \in X$.

**Theorem 2.11.** Let $(X,M,N,*,\lozenge)$ be a complete intuitionistic fuzzy metric space and let A, B, Sand T be self-mappings of X satisfying the following conditions:

(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,

(ii) S and T are continuous,

(iii) The pairs $\{A,S\}$ and $\{B,T\}$ are compatible mappings of type (P) on X,

(iv) There exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,

$M(Ax,By,qt) \geq M(Sx,Ty,t) * M(Ax,Sx,t) * M(Bx,Ty,t) * M(Ax,Ty,t)$

And

$N(Ax,By,qt) \geq N(Sx,Ty,t) \Diamond N(Ax,Sx,t) \Diamond N(Bx,Ty,t) \Diamond N(Ax,Ty,t)$

Then A, B, S and T have a unique common fixed point in X.

The aim of this paper is to extend Theorem 2.11 in the framework of intuitionistic fuzzy metric space.

## III. MAIN RESULTS

**Theorem 3.1.** Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric spaceand let A, B, S and T be self-mappings of X satisfying the following conditions:

(i) $A(X) \subset T(X)$, $B(X) \subset S(X)$,

(ii) S and T are continuous,

(iii) The pairs $\{A,S\}$ and $\{B,T\}$ are compatible mappings of type (P) on X,

(iv) There exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax,By,qt) \geq M(Sx,Ty,t) * M(Ax,Sx,t) * M(Bx,Ty,t) * M(Ax,Ty,t) * \left\{\frac{M(Sx,Ty,t) * M(Bx,Ty,t)}{M(Ax,Ty,t)}\right\} * \left\{\frac{M(Ax,Sx,t) * M(Sx,Ty,t)}{M(Ax,Ty,t)}\right\}.$$

and

$$N(Ax,By,qt) \leq N(Sx,Ty,t) \Diamond N(Ax,Sx,t) \Diamond N(Bx,Ty,t) \Diamond N(Ax,Ty,t) \Diamond \left\{\frac{N(Sx,Ty,t) \Diamond N(Bx,Ty,t)}{1-N(Ax,Ty,t)}\right\} \Diamond \left\{\frac{N(Ax,Sx,t) \Diamond N(Sx,Ty,t)}{1-N(Ax,Ty,t)}\right\}.$$

Then A, B, S and T have a unique common fixed point in X.

**Proof.** Since $A(X) \subset T(X)$ and $B(X) \subset S(X)$. We define a sequence $\{y_n\}$ such that

$y_{2n-1} = T x_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$, $\forall n \in N$.

We shall prove that $\{y_n\}$ is a Cauchy sequence. From (iv), we have

$$M(y_{2n+1}, y_{2n+2},qt) = M(Ax_{2n},Bx_{2n+1},qt)$$
$$\geq M(Sx_{2n},T x_{2n+1},t) * M(Ax_{2n},Sx_{2n},t) * M(Bx_{2n+1},T x_{2n+1},t) * M(Ax_{2n},T x_{2n+1},t) * \left\{\frac{M(Sx_{2n},Tx_{2n+1},t) * M(Bx_{2n+1},Tx_{2n+1},t)}{M(Ax_{2n},Tx_{2n+1},t)}\right\} * \left\{\frac{M(Ax_{2n},Sx_{2n},t) * M(Sx_{2n},Tx_{2n+1},t)}{M(Ax_{2n},Tx_{2n+1},t)}\right\}$$
$$= M(y_{2n}, y_{2n+1},t) * M(y_{2n+1}, y_{2n},t) * M(y_{2n+2}, y_{2n+1},t) * M(y_{2n+1}, y_{2n+1},t) * \left\{\frac{M(y_{2n},y_{2n+1},t) * M(y_{2n+2},y_{2n+1},t)}{M(y_{2n+1},y_{2n+1},t)}\right\} * \left\{\frac{M(y_{2n+1},y_{2n},t) * M(y_{2n},y_{2n+1},t)}{M(y_{2n+1},y_{2n+1},t)}\right\}$$

$\geq M(y_{2n}, y_{2n+1},t) * M(y_{2n+1}, y_{2n+2},t) * M(y_{2n}, y_{2n+1},t) * M(y_{2n+2}, y_{2n+1},t)$

which implies

$M(y_{2n+1}, y_{2n+2},qt) \geq M(y_{2n}, y_{2n+1},t) * M(y_{2n+1}, y_{2n+2},t)$.

Similarly, we have

$M(y_{2n+2}, y_{2n+3},qt) \geq M(y_{2n+1}, y_{2n+2},t)$.

Hence, we have

$M(y_{n+1}, y_{n+2},qt) \geq M(y_n, y_{n+1},t)$. (1)

Now

$$N(y_{2n+1}, y_{2n+2},qt) = N(Ax_{2n},Bx_{2n+1},qt)$$
$$\geq N(Sx_{2n},T x_{2n+1},t) \Diamond N(Ax_{2n},Sx_{2n},t) \Diamond N(Bx_{2n+1},Tx_{2n+1},t) \Diamond N(Ax_{2n},Tx_{2n+1},t) \Diamond \left\{\frac{N(Sx_{2n},Tx_{2n+1},t) \Diamond N(Bx_{2n+1},Tx_{2n+1},t)}{1-N(Ax_{2n},Tx_{2n+1},t)}\right\} \Diamond \left\{\frac{N(Ax_{2n},Sx_{2n},t) \Diamond N(Sx_{2n},Tx_{2n+1},t)}{1-N(Ax_{2n},Tx_{2n+1},t)}\right\}$$
$$= N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n},t) \Diamond N(y_{2n+2}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n-1},t) \Diamond \left\{\frac{N(y_{2n},y_{2n+1},t) \Diamond N(y_{2n+2},y_{2n+1},t)}{1-N(y_{2n+1},y_{2n+1},t)}\right\} \Diamond \left\{\frac{N(y_{2n+1},y_{2n},t) \Diamond N(y_{2n},y_{2n+1},t)}{1-N(y_{2n+1},y_{2n+1},t)}\right\}$$

$\geq N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n+2},t) \Diamond N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+2}, y_{2n+1},t)$

which implies

$N(y_{2n+1}, y_{2n+2},qt) \geq N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n+2},t)$.

Similarly, we have $N(y_{2n+2}, y_{2n+3},qt) \geq N(y_{2n+1}, y_{2n+2},t)$.

Hence, we have

$N(y_{n+1}, y_{n+2},qt) \geq N(y_n, y_{n+1},t)$. (2)

Equations (1) and (2) show that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, $\{y_n\}$ converges to some point $z \in X$ and so sequences

$\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converge to z.

Then, we have

$AAx_{2n-2} \to Sz$ and $SSx_{2n} \to Az$, (3)

and

*$BBx_{2n-1} \to Tz$ and $TT x_{2n-1} \to Bz$. (4)*

From (iv), we get

$$M(AAx_{2n-2},BBx_{2n-1},qt) \geq M(SAx_{2n-2},TBx_{2n-1},t) * M(AAx_{2n-2},SAx_{2n-1},t) * M(BBx_{2n-1},TBx_{2n-1},t) * M(AAx_{2n-2},TBx_{2n-1},t) * \left\{\frac{M(SA_{2n-2},TB_{2n-1},t) * M(BB_{2n-1},TB_{2n-1},t)}{M(AA_{2n-2},TB_{2n-1},t)}\right\} * \left\{\frac{M(AA_{2n-2},SA_{2n-1},t) * M(SA_{2n-2},TB_{2n-1},t)}{M(AA_{2n-2},TB_{2n-1},t)}\right\}$$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$$M(Sz,Tz,qt) \geq M(Sz,Tz,t) * M(Sz,Sz,t) * M(Tz,Tz,t) * M(Sz,Tz,t) * \left\{\frac{M(Sz,T z,t) * M(T z,T z,t)}{M(Sz,T z,t)}\right\} * \left\{\frac{M(Sz,Sz,t) * M(Sz,T z,t)}{M(Sz,T z,t)}\right\}$$

$\geq M(Sz,Tz,t) * 1 * 1 * M(Sz,Tz,t) * 1 * 1$

$\geq M(Sz,Tz,t)$

implies

$M(Sz, Tz,qt) \geq M(Sz,Tz,t)$.

Similarly,

$N(AAx_{2n-2},BBx_{2n-1},qt) \geq N(SAx_{2n-2},TBx_{2n-1},t) \Diamond N(AAx_{2n-2},SAx_{2n-1},t) \Diamond N(BBx_{2n-1},TBx_{2n-1},t) \Diamond N(AAx_{2n-2},TBx_{2n-1},t) \Diamond \{\frac{N(SA_{2n-2},TB_{2n-1},t) \Diamond N(BB_{2n-1},TB_{2n-1},t)}{1-N(AA_{2n-2},TB_{2n-1},t)}\} \Diamond \{\frac{N(AA_{2n-2},SA_{2n-1},t) \Diamond N(SA_{2n-2},TB_{2n-1},t)}{1-N(AA_{2n-2},TB_{2n-1},t)}\}$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$N(Sz,Tz,qt) \geq N(Sz,Tz,t) \Diamond N(Sz,Sz,t) \Diamond N(Tz,Tz,t) \Diamond N(Sz,Tz,t) \Diamond \{\frac{N(Sz,Tz,t) \Diamond N(T z,T z,t)}{1-N(Sz,T z,t)}\} \Diamond \{\frac{N(Sz,Sz,t) \Diamond N(Sz,Tz,t)}{1-N(Sz,T z,t)}\}$

$\geq N(Sz,Tz,t) \Diamond 0 \Diamond 0 \Diamond N(Sz,Tz,t) \Diamond N(Sz, Tz,t) \Diamond N(Sz, Tz,t)$

$\geq N(Sz,Tz,t)$

implies

$N(Sz,Tz,qt) \geq N(Sz,Tz,t)$.

It follows that

$Sz = T z$. (5)

Now, again from (iv), we have

$M(Az,BTx_{2n-1},qt) \geq M(Sz, TTx_{2n-1},t) \ast M(Az,Sz,t) \ast M(BTx_{2n-1},TTx_{2n-1},t) \ast M(Az,TTx_{2n-1},t) \ast \{\frac{M(Sz,TT_{2n-1},t) \ast M(BT_{2n-1},TT_{2n-1},t)}{M(Az,TT_{2n-1},t)}\} \ast \{\frac{M(Az,Sz,t) \ast MSz, TT_{2n-1},t)}{M(Az,TT_{2n-1},t)}\}$.

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$M(Az, Tz,qt) \geq M(Sz,Sz,t) \ast M(Az, Tz,t) \ast M(Tz, Tz,t) \ast M(Az, Tz,t) \ast \{\frac{M(Sz,Tz,t) \ast M(Tz,T z,t)}{M(Az,Tz,t)}\} \ast \{\frac{M(Az,Sz,t) \ast M(Sz,Tz,t)}{M(Az,Tz,t)}\}$

$\geq 1 \ast M(Az, Tz,t) \ast 1 \ast M(Az, Tz,t)$

$\geq M(Az,Tz,t)$

Implies

$M(Az,Tz,qt) \geq M(Az,Tz,t)$.

Similarly,

$N(Az,BT x_{2n-1},qt) \geq N(Sz,TT x_{2n-1},t) \Diamond N(Az,Sz,t) \Diamond N(BT x_{2n-1},TT x_{2n-1},t) \Diamond N(Az,TT x_{2n-1},t) \Diamond \{\frac{N(Sz,TT_{2n-1},t) \Diamond N(BT_{2n-1},TT_{2n-1},t)}{1-N(Az,TT_{2n-1},t)}\} \Diamond \{\frac{N(Az,Sz,t) \Diamond N(Sz,TT_{2n-1},t)}{1-N(Az,TT_{2n-1},t)}\}$.

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$N(Az,Tz,qt) \geq N(Sz,Sz,t) \Diamond N(Az,Tz,t) \Diamond N(Tz,Tz,t) \Diamond N(Az,Tz,t) \Diamond \{\frac{N(Sz,Tz,t) \ast N(Tz,Tz,t)}{1-N(Az,Tz,t)}\} \Diamond \{\frac{N(Az,Sz,t) \ast N(Sz,Tz,t)}{1-N(Az,Tz,t)}\}$

$\geq 0 \Diamond N(Az,Tz,t) \Diamond 0 \Diamond N(Az, Tz,t) \Diamond 0 \Diamond N(Az, Tz,t)$

$\geq N(Az, Tz,t)$

Implies $N(Az, Tz, qt) \leq N(Az, Tz,t)$.

It follows that

$Az = Tz$. (6)

Now from (iv) and using (5) and (6), we have

$M(Az, Bz, qt) \geq M(Sz, Tz,t) \ast M(Az,Sz,t) \ast M(Bz, Tz,t) \ast M(Az, Tz,t) \{\frac{M(Sz,Tz,t) \ast M(B z,Tz,t)}{M(Az,Tz,t)}\} \ast \{\frac{M(Az,Sz,t) \ast M(Sz,Tz,t)}{M(Az,Tz,t)}\}$

$= M(Az,Az,t) \ast M(Az,Az,t) \ast M(Bz,Az,t) \ast M(Az,Az,t)$

$\geq 1 \ast 1 \ast M(Bz,Az,t) \ast 1 \ast M(Bz,Az,t) \ast 1$

$\geq M(Az,Bz,t)$

Implies

$M(Az,Bz,qt) \geq M(Az,Bz,t)$.

Similarly,

$N(Az,Bz,qt) \geq N(Sz, Tz,t) \Diamond N(Az,Sz,t) \Diamond N(Bz, Tz,t) \Diamond N(Az, Tz,t) \{\frac{N(Sz,T z,t) \Diamond N(B z,T z,t)}{1-N(Az,T z,t)}\} \Diamond \{\frac{N(Az,Sz,t) \Diamond N(Sz,T z,t)}{1-N(Az,T z,t)}\}$

$= N(Az,Az,t) \Diamond N(Az,Az,t) \Diamond N(Bz,Az,t) \Diamond N(Az,Az,t)$

$\geq 0 \Diamond 0 \Diamond N(Bz,Az,t) \Diamond 0 \Diamond N(Bz,Az,t) \Diamond 0$

$\geq N(Az,Bz,t)$

Implies

$N(Az,Bz,qt) \leq N(Az,Bz,t)$.

It follows that

$Az = Bz$. (7)

From (5), (6) and (7), we have

$Az = Bz = T z = Sz$.

Now, we shall show that $Bz = z$.

Again from (iv), we have

$M(Ax_{2n},BT z,qt) \geq M(Sx_{2n},Tz,t) \ast M(Ax_{2n},Sx_{2n},t) \ast M(BTz,Tz,t) \ast M(z, Tz,t) \ast \{\frac{M(Sx_{2n},Tz,t) \ast M(BTz,Tz,t)}{M(z,Tz,t)}\} \ast \{\frac{M(Ax_{2n},Sx_{2n},t) \ast M(Sx_{2n},Tz,t)}{M(z,Tz,t)}\}$.

Using (5) and (6) and taking the limit as $n \to \infty$, we have

$M(z,Bz,qt) \geq M(z, Tz,t) \ast M(z,z,t) \ast M(Bz,Tz,t) \ast M(z,Tz,t) \{\frac{M(z,T z,t) \ast M(B z,T z,t)}{M(z,T z,t)}\} \ast \{\frac{M(z,z,t) \ast M(z,T z,t)}{M(z,T z,t)}\}$

$= M(z,Bz,t) \ast 1 \ast M(Az,Az,t) \ast M(z,Bz,t) \ast 1 \ast 1$

$\geq M(z,Bz,t)$

implies

$M(z,Bz,qt) \geq M(z,Bz,t)$.

Similarly,

$N(Ax_{2n},BTz,qt) \geq N(Sx_{2n},Tz,t) \Diamond N(Ax_{2n},Sx_{2n},t) \Diamond N(BTz,T z,t) \Diamond N(z, Tz,t) \Diamond \{\frac{N(Sx_{2n},Tz,t) \Diamond N(BTz,Tz,t)}{1-N(z,Tz,t)}\} \Diamond \{\frac{N(Ax_{2n},Sx_{2n},t) \Diamond N(Sx_{2n},Tz,t)}{1-N(z,Tz,t)}\}$.

Using (5) and (6) and taking the limit as n → ∞, we have

N(z,Bz,qt) ≥ N(z, Tz,t)◊N(z,z,t)◊ N(Bz,Tz,t)◊N(z, Tz,t)◊

$\{\frac{N(z,Tz,t)\Diamond N(Bz,Tz,t)}{1-N(z,Tz,t)}\}\Diamond\{\frac{N(z,z,t)\Diamond N(z,Tz,t)}{1-N(z,Tz,t)}\}$

= N(z,Bz,t) ◊ 0◊ N(Az,Az,t) ◊N(z,Bz,t)◊N(z,Bz,t)◊N(z,Bz,t)

≥ N(z,Bz,t) Implies

N(z,Bz,qt) ≤ N(z,Bz,t).

It follows that

Bz = z. (8)

Thus from (8), z = Az = Bz =Tz = Sz and hence z is a common fixed point of the mappings A, B, S and T.

Uniqueness,

Let w be another common fixed point of A, B, S and T. Then

M(z,w,qt) = M(Az,Bw,qt)

≥ M(Sz,Tw,t) ∗ M(Az,Sz,t) M(Bw,Tw,t) ∗M(Az,Tw,t)

$\{\frac{M(Sz,Tw,t)* M(Bw,Tw,t)}{M(Az,Tw,t)}\}\{\frac{M(Az,Sz,t)* M(Sz,Tw,t)}{M(Az,Tw,t)}\}$

≥ M(z,w,t)

Implies

M(z,w,qt) ≥ M(z,w,t).

And

N(z,w,qt) = N(Az,Bw,qt)

≤ N(Sz,Tw,t)◊ N(Az,Sz,t)◊N(Bw,Tw,t)◊ N(Az,Tw,t)

$\{\frac{N(Sz,Tw,t)\Diamond N(Bw,Tw,t)}{1-N(Az,Tw,t)}\}\Diamond\{\frac{N(Az,Sz,t)\Diamond N(Sz,Tw,t)}{1-N(Az,Tw,t)}\}$

≤ N(z,w,t)

Implies

N(z,w,qt) ≤ N(z,w,t).

Hence z = w. This completes the proof.

Corollary 3.2. Let (X, M, N, , ◊) be a complete intuitionistic fuzzy metric space and let A, B, S and T be self- mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists q  (0,1) such that for all x, y  X andt > 0,

M(Ax,By,qt)

≥ M(Sx,Ty,t) ∗M(Ax,Sx,t) ∗M(By,Ty,t) ∗M(By,Sx,2t) M(Ax,Ty,t)

$\{\frac{M(Sx,Ty,t)* M(By,Ty,t)}{M(Ax,Ty,t)}\}\{\frac{M(Ax,Sx,t)* M(Sx,Ty,t)}{M(Ax,Ty,t)}\}$

and

N(Ax,By,qt)

≤ N(Sx,Ty,t)◊ N(Ax,Sx,t)◊ N(By,Ty,t)◊ N(By,Sx,2t)◊ N(Ax,Ty,t)◊

$\{\frac{N(Sx,Ty,t)\Diamond N(By,Ty,t)}{1-N(Ax,Ty,t)}\}\Diamond\{\frac{N(Ax,Sx,t)\Diamond N(Sx,Ty,t)}{1-N(Ax,Ty,t)}\}$

Then A, B, S and T have a unique common fixed point in X.

**Corollary 3.3.** Let (X, M, N, ,◊ ) be a complete intuitionistic fuzzy metric space and let A, B, S and T be self- mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists q  (0,1) such that for all x, y  X and t > 0, M(Ax,By,qt) ≥ M(Sx,Ty,t) and N(Ax,By,qt) ≤ N(Sx,Ty,t).

Then A, B, S and T have a unique common fixed point in X.

**Corollary 3.4.** Let (X, M, N, , ◊) be a complete intuitionistic fuzzy metric space and let A, B, S and T be self- mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists q  (0,1) such that for all x, y  X and t> 0,

M(Ax,By,qt) ≥ M(Sx,Ty,t)  M(Sx,Ax,t)  M(Ax,Ty,t) and

N(Ax,By,qt) ≤ N(Sx,Ty,t)◊ N(Sx,Ax,t)◊ N(Ax,Ty,t).

Then A, B, S and T have a unique common fixed point in X.

**Theorem 3.5.** Let (X,M,N,,◊) be a complete intuitionistic fuzzy metric space. If S and T are continuous self- mappings of X, then mappings S and T havea common fixed point in X if and only if there exists a self- mapping A of X satisfying the following conditions:

(i) A(X)  T(X)∩S(X),

(ii) the pairs {A,S} and {A,T} are compatible mappings of type (P) on X,

(iii) there exists q  (0,1) such that for all x, y  X and t > 0,

M(Ax,Ay,qt) ≥ M(Sx,Ty,t)  M(Ax,Sx,t) M(Ay,Ty,t)  M(Ax,Ty,t)

and

N(Ax,Ay,qt) ≤ N(Sx,Ty,t)◊ N(Ax,Sx,t)◊ N(Ay,Ty,t)◊ N(Ax,Ty,t).

Then A, S and T have a unique common fixed point in X.

Proof. Necessary part. Let S and T have a common fixed point in X, say z, then Sz = z = Tz. Let Ax = z for all x  X, then A(X)  T(X)∩S(X) and we know that {A,S} and {A,T} are compatible mappings of type (P), in fact A∘S = S∘A and A ∘ T = T ∘ A and hence the conditions (i) and (ii) are satisfied. For someq  (0,1),we have M(Ax,Ay,qt) =1 ≥ M(Sx,Ty,t)  M(Ax,Sx,t)  M(Ay,Ty,t)  M(Ax,Ty,t)

and
N(Ax,Ay,qt) = 0 ≤ N(Sx,Ty,t)◊ N(Ax,Sx,t)◊ N(Ay,Ty,t)◊ N(Ax,Ty,t)
for all x, y ∈ X and t > 0. Hence the condition (iii) is satisfied.

**Sufficient part.** Let A = B in Theorem 3.1. Then A, S and T have a unique
common fixed point in X. This completes the proof.

**Corollary 3.6.** Let (X, M, N, ∗,◊ ) be a complete intuitionistic fuzzy metric
space. If S and T are continuous self-mappings of X, then mappings S and T have a common fixed point in X if and only if there exists a self-mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists q ∈ (0,1)such that for all x, y ∈ X and t > 0,
M(Ax,Ay,qt)
≥ M(Sx,Ty,t) ∗ M(Ax,Sx,t) ∗ M(Ay,Ty,t) ∗ M(Ax,Sx,2t) ∗ M(Ax,Ty,t) and
N(Ax,Ay,qt)
≤ N(Sx,Ty,t)◊ N(Ax,Sx,t)◊ N(Ay,Ty,t)◊ N(Ax,Sx,2t)◊ N(Ax,Ty,t).
Then A, S and T have a unique common fixed point in X.

**Corollary 3.7.** Let (X, M, N, ∗, ◊) be a complete intuitionistic fuzzy metric
space. If S and T are continuous self-mappings of X, then mappings S and
T have a common fixed point in X if and only if there exists a self-mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists q ∈ (0,1)such that for all x, y ∈ X and t > 0,
M(Ax,Ay,qt) ≥ M(Sx,Ty,t) and N(Ax,Ay,qt) ≤ N(Sx,Ty,t).
Then A, S and T have a unique common fixed point in X.

**Corollary 3.8**. Let (X, M, N, ∗,◊ ) be a complete intuitionistic fuzzy metric
space. If S and T are continuous self mappings of X, then mappings S and
T have a common fixed point in X if and only if there exists a self mapping Aof X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists q ∈ (0,1)such that for all x, y ∈ X and t > 0,
M(Ax,Ay,qt) ≥ M(Sx,Ty,t) ∗ M(Sx,Ax,t) ∗ M(Ax,Ty,t)
And
N(Ax,Ay,qt) ≤ N(Sx,Ty,t)◊ N(Ax,Sx,t) ◊N(Ax,Ty,t).
Then A, S and T have a unique common fixed point in X.

**Example 3.9.** Let X = {1/n: n ∈ N}∪{0} with ∗ continuous t-norm and continuoust-conorm defined by a ∗ b = ab and a◊b = min{1,a+b} respectively, for a, b ∈ [0,1]. For each t ∈ [0,∞) and x, y ∈ X, define (M,N) by

M(x, y,t) = $\frac{t}{t+|x-y|}$ if t > 0,

=0, if t = 0.

And

N(x, y,t) = $\frac{|x-y|}{t+|x-y|}$ if t > 0,

=1, if t = 0.

Clearly (X, M, N, ∗, ◊) is an intuitionistic fuzzy metric space.
Define A(x) = B(x) = x6and S(x) = T(x) = $x^2$on X. It is clear that A(X) ⊆T(X) and B(X) ⊆ S(X).
Now

M(Ax,By,t/3) $=\frac{t/3}{\frac{t}{3}+\frac{|x-y|}{6}}=\frac{2t}{2t+|x-y|}\geq\frac{t}{t+\frac{|x-y|}{2}}$ =M (T x,Sy,t),

And

N(Ax,By,t/3) $=\frac{|x-y|/6}{\frac{t}{3}+\frac{|x-y|}{6}}=\frac{|x-y|}{2t+|x-y|}\geq\frac{|x-y|}{t+\frac{|x-y|}{2}}$=N (T x,Sy,t).

Thus all the conditions of Theorem 3.1 are satisfied and so A, B, S and T have a unique common fixed point.

## REFERENCES


[1] C. Alaca - D. Turkoglu - C. Yildiz, Fixed points in intuitionstic fuzzy metricspaces, Chaos, *Solutions and Fractals* **29** (2006), 1073–1078.

[2] C. Alaca, On fixed point theorems in intuitionistic fuzzy metric spaces, Commun. *Korean Math. Soc.* **24** (4) (2009), 565–579.

[3] L. B. Ciri ´ c - S. N. Je ´ si˜ c - J. S. Ume, ´ The existence theorems for fixed and periodic points of non expansive mappings in intuitionistic fuzzy metric spaces, Chaos, *Solitone and Fractals*, **32** (2008), 781–791.

[4] Y. J. Cho - B. K. Sharma - D. R. Sahu, Semi-compatibility and fixed points, *Math. Japonica* **42** (1) (1995), 91–98.

[5] A. George - P. Veeramani, On some results in fuzzy metric spaces, *Fuzzy Sets and Systems* **64** (1994), 395–399.

[6] A. George - P. Veeramani, On some results of analysis for fuzzy metric spaces, Fuzzy Sets and Systems 90 (1997), 365–368.

[7] V. Gregori - A. Sapena, On fixed point theorems in fuzzy metric spaces, *Fuzzy Sets and Systems* **125** (2002), 245–252.

[8] G. Jungck, Compatible mappings and common fixed points, *Internat J. Math. Math. Sci.* **9** (4) (1986), 771–779.

[9] G. Jungck - P. P. Murthy - Y. J. Cho, Compatible mappings of type (A) and common fixed points, *Math. Japonica* **36** (1993), 381–390.

[10] M. Koireng - Y. Rohen, Common fixed point theorems of compatible mappings of type (P) in fuzzy metric spaces, *Int. J. Math. Anal.* **6** (4) (2012), 181–188.

[11] I. Kramosil - J. Michalek, Fuzzy metrics and statistical metric spaces, *Kybernetika* **11** (5) (1975), 336–344.
[12] A. Mohamad, Fixed point theorems in intuitionistic fuzzy metric spaces, *Chaos, Solutions and Fractals* **34** (2007), 1689–1695.
[13] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, *Solutions and Fractals* **22**(2004), 1039–1046.
[14] H. K. Pathak - S. S. Chang - Y. J. Cho, Fixed point theorems for compatible mappings of type (P), *Indian J. Math*. **36** (2) (1994), 151–166.
[15] H. K. Pathak - Y. J. Cho - S. M. Kang - B. S. Lee, Fixed point theorems for compatible mappings of type (P) and applications to dynamic programming, *Le Matematiche* **50** (1) (1995), 15–33.
[16] H. K. Pathak - S. M. Kang, Compatible mappings of type(B) and common fixed point theorems of Gregus type ˇ , *Czechoslovak Math. J.* **45** (120) (1995), 685–698.
[17] H. K. Pathak - Y. J. Cho - S. M. Kang - B. Madharia, Compatible mappings of type (C) and common fixed point theorem of Gregus type, *Demonstr. Math.* **31** (3)(1998), 499–517.
[18] S. Romaguera - P. Tirado, Contraction maps on Ifqm-spaces with application withrecurrence equations of quicksort, *Electronic Notes in Theoretical Computer Science*, **225** (2009), 269–279.
[19] B. Schweizer - A. Sklar, Statistical spaces, Pacific J. Math. 10 (1960), 313–334.
[20] S. Sharma - S. Kutukcu - R. S. Rathore, Common fixed point for multivalued mappingsin intuituinstic fuzzy metric spaces, Commun. Korean Math. Soc. 22 (3)(2007), 391–399.
[21] B.P. Tripathi, G.S. Saluja, D.P. Sahu and N. Namdeo,Common fixed point theorems of compatible mapping of type (p) in intuitionistic fuzzy metric spaces, *LE Matematiche* **LXIX**(2014),77-92.
[22] D. Turkoglu - C. Alaca - C. Yildiz, Compatible maps and compatible of type (α) and (β) in intuituinstic fuzzy metric spaces, *Demonstratio Math.* **39** (3) (2006), 671–684.
[23] D. Turkoglu - C. Alaca - Y. J. Cho - C. Yildiz, Common fixed point theorems in intuitionistic fuzzy metric spaces, *J. Appl. Math. Comput.* **22** (1-2) (2006), 411–424.
[24] L. A. Zadeh, Fuzzy Sets, *Information and Control* **8** (1965), 338–353.